A Methodology for Estimation of Bus Dwell Time and Prediction Intervals
- A Case Study of Houston Metro -
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ABSTRACT: The variability in bus dwell time is the key issues in advanced traveler information system (ATIS) and transit signal priority (TSP) systems because the successful estimation of the bus arrival time is greatly dependent on the accurate prediction of the bus dwell time at stops. In this paper, a probabilistic model and the weighted least squares (WLS) regression model were developed to predict the bus dwell time. Although the probabilistic model showed the strength in representing the determinants of dwell time as random variables, the observed data did not support the model assumption of the linear correlation between the number of alighting passengers and the passenger loads. It was concluded from an analysis of the dwell time determinants that the bus headway is most significant determinant and that the regression model with the bus headway as an independent variable gave the best results. Because of the non-constant variance over bus headways, the weighted least squares method was employed in order to consider the non-constant variance. Another advantage of the WLS method is that the prediction interval may be readily calculated. The prediction interval is more informative than mean dwell time in supporting ATIS and TSP systems.

Key Words: bus dwell time, dwell time determinants, weighted least squares, prediction interval

요약: 첨단여행자정보체계와 대중교통우선신호체계의 성패는 얼마나 버스도착시간을 정확하게 예측하는가에 달려있다. 버스도착시간은 버스정차시간에 의해 크게 영향을 받지만 버스정차시간의 불확실성으로 인해 그 예측이 매우 어렵다. 본 연구에서는 확률모형과 가중회귀분석모형을 사용하여 버스 정차시간을 예측하였다. 확률모형은 버스정차시간에 영향을 미치는 요인들을 확률적으로 설명할 수 있는 장점이 있지만 요인간의 선형상관관계를 바탕으로 하기 때문에 이 가정이 만족되지 않는 상황에서는 적용이 불가능하다. 이러한 한계를 극복하기 위해 가중회귀분석을 활용하여 정차시간을 예측하는 방법과 정차시간의 예측구간을 추정하는 방법론을 제시하였다.

주제어: 버스정차시간, 버스정차시간 영향요인, 가중회귀분석, 예측구간

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1. Introduction

One of the key issues in advanced traveler information system (ATIS) and transit signal priority (TSP) system is the variability in bus arrival times at bus stops or intersections. In order to accurately predict bus arrival time at intersections with nearside bus stops, it is imperative that the bus dwell time be considered explicitly. The bus dwell time depends on several characteristics of the site and the bus operations including the passenger demand at stop, the frequency of the bus service, the schedule adherence, the number of bus doors, the type of the fare collection system, and other factors. In the context of advanced traveler information system, the dwell time is treated as a component of the bus travel time and predicted in conjunction with the link travel time for entire bus routes or a section of the route. Accurate bus arrival information can be used in scheduling passengers’ departure times and coordinating transfers (Chien et al., 2002). However, it has been a challenging task to predict the dwell time at the level of the individual bus due to its relatively large variability that is caused by the randomness in passenger demand and the variation in the service time for each passenger.

In this study, a probabilistic model is employed for the representation of the stochastic nature of dwell time. This is because probabilistic models consider all determinants as random variables and use the probability density functions as input requirements. A weighted least squares (WLS) regression model was also developed to model the situation where the variance of dwell time is not constant. WLS regression models are known to be useful for estimating the values of model parameters when the response values have differing degrees of variability over the values of the independent variables, which violates the common assumption of the constant variance for regression modeling. The forecasting process, including the prediction model for dwell time, is itself subject to some prediction error. Moreover, the large variations in dwell times introduce additional errors in the prediction model. In order to accommodate the variability in dwell time, the prediction error bound (i.e. prediction interval) is obtained from the WLS regression model. The prediction interval is more informative than mean dwell time in supporting ATIS and TSP systems. Kim and Rilett (2005) showed that TSP system incorporating the prediction interval significantly improves bus operations at the intersections with nearside bus stops. Therefore, the objective of this study is to develop a methodology for predicting the mean dwell time, and the associated prediction interval.

The next section of this paper discusses the characteristics of the study site and the dwell time data. It is followed by a development of the dwell time prediction model based on the stochastic nature of passenger demand. The most significant determinant is selected by statistical methods and a weighted least square model is developed in order to estimate the bus dwell time as well as its associated prediction interval. The final section of the paper discusses the conclusions.
II. Background

The Highway Capacity Manual (TRB, 2000) defines the bus dwell time as the amount of time that a bus spends while stopped to serve passengers at specific stop. It is the time required to serve passengers at the busiest door, plus the time required to open and close the doors. In the absence of other information, the Highway Capacity Manual recommends that dwell time can be assumed to be 60 seconds for central business districts (CBD), transit centers, major on-line transfer points, or major park-and-ride stops; 30 seconds for major outlying stops; and 15 seconds for typical outlying stops. Levinson (1983) found that dwell time ranges from 20 to 60 seconds in the CBD, and 10 to 15 seconds for non-CBD stops. He also found that dwell time can account for up to 26 percent of the total travel time. Dwell time has been generally accepted as a major factor causing vehicle bunching, which in turn results in large variability of headways (Rajbhandari et al., 2003). Significant headway variation may result in longer average passenger waiting times and congested passenger loads, both of which degrade the quality of the transit service.

A common MOE of bus dwell time is the coefficient of variation (C.V.) of dwell times, which is the standard deviation of dwell time observations divided by the mean. In several U.S. cities, the C.V. of dwell times typically ranges from 40 to 80 percent (TRB, 2000).

Most models proposed for estimating dwell time have been based on the linear relationship between the number of alighting/boarding passengers and dwell time. Several ordinary least squares (OLS) regression models have been proposed for relating dwell time to passenger demand, specifically boarding and alighting passengers (Kraft and Bergen 1974; Lin and Wilson 1992). The regression analyses found that the marginal service time is affected by various factors such as the time of day, fare collection system, and the number of passenger standing. With the popularization of Automatic Passenger Counter (APC) among transit agents in U.S. since 1990's, a rich set of dwell time observations can be collected at the level of individual bus stops (Kimpel et al., 2003). In addition, the large quantity of data allows analysis of rare events, such as lift operations (Dueker et al., 2004). The literature on regression modeling of dwell time indicates that the most significant determinants of dwell time are number of boarding passengers, number of alighting passengers, and number of passenger standing. It was also found that marginal service times for boarding and alighting movements were 4 seconds and 2 seconds, respectively, greater than HCM values.

The application of these types of regression models, however, is limited to the situation when the exact number of alighting/boarding passengers can be observed or have been observed.

Some research has been conducted on representing the number of passengers as independent random variables with given density functions. Powell and Sheffi (1983) derived probability distributions of alighting and boarding times for one door and two door buses. They found that Binomial distribution and Poisson distribution are most natural representation of the number of alighting passenger and the number of boarding passenger, respectively. Guenthner and
Sinha (1983) found that negative binomial distribution was an acceptable descriptor of the number of boarding and alighting passenger, especially on higher demand routes. Adamski (1992) proposed probabilistic models of passenger service processes at bus stops. He developed three density functions for boarding/alighting times based on the Exponential, Gamma, and Erlangian probability distributions. The means and standard deviations of dwell time were calculated using moment functions for each density function. The comparison with real-world observations indicated that the approximation by each distribution obtained quite good estimations for the mean and the standard deviation of dwell time.

### III. Study Site and Data Collection

Three Metro bus stops located along the eastbound of Bellaire Boulevard in Houston, Texas were selected as the study site as shown in Figure 1. The Bellaire Blvd is a heavily traveled cross town arterial primarily traversing high-density residential areas.

The southern area of Bellaire Blvd is mainly occupied by single family residences. In contrast, in the northern area, multiple family residences are dominant. Three regular bus routes operate on the study site in Figure 1. The service frequency (i.e. bus headway) of each route ranges from 6 minutes to 40 minutes according to the time of day. The obtained data included number of alighting/boarding passengers, passenger loads, and bus dwell times. The data were collected by placing observers at each bus stop and all observers were equipped with digital timers synchronized to a reference time. The data collection was performed between 7:00 AM and 10:00 AM on five weekdays in 2003. A total of 217 buses were observed during the entire data collection period: 35 observations of route #17, 67 observations of route #163, and 115 observations of route #2. The average bus dwell time and its standard deviation for each bus stop are summarized in Table 1. Dwell times during off-peak period were longer than those during the peak period because of longer scheduled headways. However, the coefficients of variation (C.V.) were slightly higher during peak period because of correspondingly the larger variation in passenger arrivals. The values of C.V. for all stop were higher than the HCM suggested value of 0.6. The variance in the average number of passenger arrivals during a given 5 minute-period was 2.0 minutes and 1.59 minutes during the peak and off-peak periods, respectively.

The information regarding each bus' adherence to its schedule was obtained by comparing the observed arrival times and the scheduled arrival times at each stop. It was decided that a bus is on schedule when it arrives at a specific stop within ± 2 minutes of its scheduled arrival time. The analysis revealed that 33 percent of buses were on schedule during the entire study period. Approximately 36 percent of the buses arrived at the stops more than 5 minute late. Figure 2 shows the difference between actual arrival time and scheduled arrival time. It can be seen that a higher percentages of buses arrive on schedule during the peak period that during the off-period. This might be because all bus routes are crossing major arterial before reaching the study site, and the delay at the intersections
Figure 1. Study site and bus route
Ⅲ. Effects of upstream activities during peak period on schedule adherence

Upstream of the study site during peak period may affect the schedule adherence of buses at the stops of interest during off-peak periods.

Ⅳ. Probabilistic Model for Estimating Dwell Time

The bus dwell time consists of the time needed for the following events to occur: 1) passenger boarding, 2) passenger alighting, and 3) schedule slack time. The schedule slack time is the time that the bus waits until its scheduled departure time. Note that the first two activities are sequential or simultaneous while event 3 occurs after both event 1 and 2 are completed. Because the field observation found that only two buses at only one stop waited with their doors open until the scheduled departure time, it was decided that the scheduled slack time would not be considered in the models developed in this paper. The probabilistic model, therefore, has been defined as a function of passenger alighting time and passenger boarding time. If the numbers of passengers alighting and boarding are independent of each other, the dwell time was expressed by the sum of two random variables as shown in Equation (1). The marginal passenger alighting and boarding time were assumed constant.

\[ t_d = N_a \cdot t_a + N_b \cdot t_b \]  

(1)
where

\[ t_d = \text{bus dwell time (sec)}, \]
\[ N_a = \text{alighting passengers per bus (pass)}, \]
\[ t_a = \text{marginal passenger alighting time (sec/pass)}, \]
\[ N_b = \text{boarding passengers per bus (pass)}, \]
\[ t_b = \text{marginal passenger boarding time (sec/pass)}. \]

The number of boarding passengers for a specific bus is equal to the number of passenger arrivals during the time between successive buses at a specific bus stop. It is assumed that all passengers for a given bus route that arrive in the time points between two buses will get aboard the first available bus for that route. Previous studies (Adamski 1992; Powell and Sheffi 1983; Chien et al., 2002) suggested that the Poisson distribution is an appropriate probability distribution for passenger arrivals at stops during the peak period. It can be seen that for the test bed the number of passenger arrivals during a specific time period (i.e. 5-minute interval) had large variation. The statistics for the passenger arrivals at each stop are summarized in <Table 2> where the units are a number of passenger arrivals during a 5-minute period.

As shown in <Table 2>, the large variation in passenger arrivals led to the employment of the negative binomial distribution. This Probability Density Function (PDF) may be characterized as having a variance higher than the mean whereas Poisson distribution requires a variance equal to the mean. The PDF of the negative binomial distribution for a random variable \( X \) can be calculated according to the following equations:

\[
P(X = x | r, p) = \binom{x + r - 1}{x} p^r (1 - p)^x\tag{2}
\]
\[
\mu = \frac{r(1 - p)}{p} \tag{3}
\]
\[
s^2 = \frac{r(1 - p)}{p^2} \tag{4}
\]

where

\( X = \) number of passenger arrival,
\( r = \) number of success,
\( p = \) probability of success,
\( \mu = \) sample population mean, and
\( s^2 = \) sample population variance.

From Equation (3) and (4), the parameters \( P \) and \( r \) can be derived based on the mean and variance of the sample population as follows:

\[
p = \frac{\mu}{s^2} \tag{5}
\]
\[
r = \frac{\mu^2}{s^2 - \mu} \tag{6}
\]

### Table 2: Summary of Statistics for Passenger Arrivals During 5 minute Period

<table>
<thead>
<tr>
<th></th>
<th>Stop A</th>
<th>Stop B</th>
<th>Stop C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (pass/5 min)</td>
<td>Variance (pass/5 min)</td>
<td>Mean (pass/5 min)</td>
</tr>
<tr>
<td>Peak (7:00 ~ 8:30)</td>
<td>1.57</td>
<td>1.60</td>
<td>1.6</td>
</tr>
<tr>
<td>Off peak (8:30 ~ 10:00)</td>
<td>1.53</td>
<td>1.87</td>
<td>0.66</td>
</tr>
</tbody>
</table>
The chi-square($X^2$) goodness of fit test was performed to evaluate the null hypothesis that the observation of passenger arrivals at each stop is drawn from assumed distribution (i.e. Poisson and negative binomial). Although the negative binomial distribution resulted in lower $X^2$ values than the Poisson distribution, there was no statistical evidence in favor of the negative binomial distribution because both distributions had not been rejected in any data set. The negative binomial distribution was selected as an appropriate descriptor for the number of passengers arriving at stop for a given time period because the underlying assumption of the Poisson distribution (i.e. equality of mean and variance) could not be held. With the assumption of stationary passenger arrivals, the negative binomial distribution can provide an expected number of passenger arrivals for various levels of bus headway. The details about this assumption can be found elsewhere (Kim, 2004).

In the case of alighting and steady-state demand conditions, a binomial distribution is known to be an appropriate representation for the number of alighting passenger (Adamski 1992; Powell and Sheffi 1983). If the number of passengers on bus is known, for example, through an APC system, the probability distribution for alighting passengers is distributed binomial $\text{Bi}(L, p)$ as shown in Equation (7).

\[
P\{\text{passenger alighting } N_a = x\} = f(x) = \sum_{L=x}^{\infty} L! \cdot (x^L \cdot x^{-x}) (1-p)^{L-x} \tag{7}
\]

where

\[
p = \text{probability of alighting for a randomly chosen passenger on bus,}
\]

\[
N_a = \text{number of passenger alighting (p), and}
\]

\[
L = \text{total number of passengers on the bus.}
\]

In order to estimate marginal passenger service time, a multiple linear regression (MLR) model was developed. The model indicated that each alighting passenger requires 2.55 second and an additional 6.07 seconds are consumed for each passenger boarding through both doors.

Based on the definition of the binomial distribution, the number of alighting passengers $N_a$ can be represented by the production of the ‘conditional expectation given passenger load $l$’ and ‘alighting probability $p$’ The number of boarding passengers can also be expressed by the conditional expectation given arrival rate $\lambda$ and bus headway $h$. Based on these substitutions, expected bus dwell time can be estimated using a model that assumes a linear relationship between passenger demand and marginal passenger service time as follows:

\[
E[t_d] = t_a \cdot E[N_a = a|L = l, P = p] + t_b \cdot E[N_b = b|A = \lambda, \Delta t = h] \tag{8}
\]

\[
t_d = t_a \cdot (p \cdot l) + t_b \cdot (\lambda \cdot h) \tag{9}
\]

where

\[
t_d = \text{bus dwell time (s)},
\]

\[
t_a = \text{marginal passenger alighting time (sec)},
\]

\[
p = \text{alighting probability for passenger in bus},
\]

\[
l = \text{passenger loads (p)},
\]

\[
t_b = \text{marginal passenger boarding time (sec/p)},
\]

\[
h = \text{bus time headway (min)},
\]

\[
\lambda = \text{passenger arrival rate (p/min)}.
\]
The alighting probability $P_k$ and passenger arrival rate $\lambda$ for each stop were estimated by Equation (10).

$$P_k = \frac{1}{n} \sum_{\text{observed alighting of bus}} \text{observed passenger load of bus}$$

(10)

where

$P_k$ = alighting probability at stop k, and

$N$ = number of observed bus at stop k.

Figure 3 shows the dwell times for each stop as a function of bus headway. The predicted dwell time showed a strong linear relation with bus headways and fluctuated moderately with variation in the number of alighting passengers. Because the dwell time itself has large variability caused by randomness in the passenger demand and passenger’s service delay, large average absolute errors were resulted as shown in <Table 3>.

Because of the large AAE values for dwell time, it was decided to conduct the diagnosis on the linear relationship between dwell time and its determinants such as alighting and boarding passengers. Through examining the significance of each determinant to dwell time, the probabilistic model proposed in this study will be evaluated and an alternative approach can be proposed, if necessary. The probabilistic model was based on

<table>
<thead>
<tr>
<th>Stop</th>
<th>AAE (sec)</th>
<th>Average Dwell Time</th>
<th>AAE (sec)</th>
<th>Average Dwell Time</th>
<th>AAE (sec)</th>
<th>Average Dwell Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>5.46 (34%)</td>
<td>953</td>
<td>3.87 (39%)</td>
<td>9.82</td>
<td>7.84 (36%)</td>
<td>21.98</td>
</tr>
<tr>
<td>Off peak</td>
<td>7.63 (37%)</td>
<td>20.46</td>
<td>3.80 (44%)</td>
<td>8.62</td>
<td>9.98 (36%)</td>
<td>27.53</td>
</tr>
<tr>
<td>Overall</td>
<td>6.35 (44%)</td>
<td>14.24</td>
<td>3.84 (41%)</td>
<td>9.33</td>
<td>8.71 (36%)</td>
<td>24.24</td>
</tr>
</tbody>
</table>

<Table 3> Average Absolute Error and Average Predicted Dwell Time
the assumption of a linear relationship between the number of alighting passengers and the number of passengers on the bus, and the number of boarding passengers and bus headways. Therefore, the probabilistic model is valid only when a linear relationship exists between the determinants.

V. Testing Significance of Determinants

A linear relationship between two random variables can be measured by the correlation coefficient (Milton and Arnold, 1995). The correlation coefficient is a measure of the degree of linear relationship between variables and it lies between -1 and 1, inclusive. High absolute value indicates strong relationship. The correlation coefficients between the number of alighting passengers and passenger loads were calculated to examine the linear relationship. The values of correlation coefficient were 0.199, 0.073, and 0.21 for stop A, B, and C, respectively. Based on these results, it can be concluded that no linear relationship exists between the number of alighting passengers and passengers on the buses at any of the stops. This is important finding because the probabilistic model is on the basis of the assumption of linear relation between alighting passengers and passenger loads. The bus headway and the number of boarding passengers showed strong linear relationship as evidenced by the value of correlation coefficient (0.726, 0.496, and 0.702 for stops A, B, and C, respectively). Although the correlation coefficient for stop B indicated only a moderate linear relationship, it can be concluded that the number of boarding passengers and bus headways has a linear relationship, in general. The correlation coefficient analysis led to the conclusion that the number of boarding passengers has a linear relationship to the bus headway while the number of alighting passengers has no linear relationship to passengers on the bus for the stops in test bed. This conclusion indicates that an assumption of the probabilistic model was violated. Therefore, an alternative method was needed for estimating dwell time. The alternative methods do not guarantee better results than the probabilistic model. However, the alternatives can provide more reliable results if they satisfy underlying modeling assumptions.

Multiple linear regression (MLR) models were developed to evaluate the statistical significance of the determinants for bus dwell time at each bus stop. Three independent variables were considered in the model: schedule adherence, passenger loads, and bus headways. These variables were selected because they have found to be related to the bus dwell time in previous studies (Kimpel et al., 2003; Dueker et al., 2004). Two sided t-tests at a 0.05 significance level were conducted to determine statistical significance for the estimated coefficients. The test results, which are shown in <Table 4>, indicate that the bus headways have a linear relationship with bus dwell time for all bus stops, but not with schedule adherence and passenger loads. Low $R^2$ values of the MLR models indicated that the observed data have large variation and the prediction with these models would results in large prediction errors.
V. Weighted Least Square Model and Prediction Interval

Simple linear models of dwell time as a function of bus headway, consequently, were developed and they are shown in Table 4. The estimated coefficient and intercept were significant at a 0.05 significance level, and the adjusted $R^2$ was similar to results of the MLR model. However, the residual analysis revealed that the variance was not constant but rather increased with increasing bus headway. A residual plot against the bus headway for stop A is shown in Figure 4. The residual plots for stop B and C also showed the funnel shaped variance over bus headways.

These results violated the assumption of constant variance in error (Neter et al., 1996), which is the underlying assumption of standard linear regression modeling. The modified Levene test for homogeneity of variance ascertained that the error terms have non-constant variance at a 0.05 significance level. Consequently a weighted least squares (WLS) regression was introduced because it has been known to be useful for estimating the values of model parameters when the response values have differing degrees of variability over the values of independent variables. In weighted least squares parameter estimation, as in regular least squares, the unknown values of the parameters in the regression function are estimated by finding the numerical values for the parameter estimates that minimize the sum of the squared deviations between the observed responses and the functional portion of
the model. Unlike least squares, however, each term in the weighted least squares criterion includes an additional weight, $W_i$, that determines how much each observation in the data set influences the final parameter estimates. The detailed information on estimation process for WLS can be found elsewhere (Carroll and Ruppert, 1988).

The results of modeling dwell time for bus stops A, B, and C are presented in Figure 5. It can be seen that there is very little difference between the weighted and unweighted lines, which indicates that they would both predict approximately the same dwell time for a given bus headway. The next step is to calculate the prediction interval of WLS model. Using the standard deviation obtained from the standard deviation function, $\sigma_0$, a 100(1 − $\alpha$)% prediction interval on a future dwell time for a given bus headway can be obtained as shown Equation (11).

$$[L_1, L_2] = \hat{d} + t_{a/2, n-2} \cdot \sigma_0 \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{2\sum x_i^2 - (\sum x_i)^2}}$$

(11)

where

- $\hat{d}$ = predicted dwell time from a single observation made when $x = x_0$ (sec),
- $t_{a/2, n-2}$ = appropriate point based on the $T_{n-2}$ distribution,
- $\sigma_0$ = standard deviation estimate from standard deviation function,
- $L_1$ = lower bound of the prediction interval, and
- $L_2$ = upper bound of the prediction interval.

The 95% prediction intervals for the dwell time response for unweighted versus weighted regression are shown in Figure 5. Because $\sigma_0$ in the WLS model increased as the predictor variable increased, the prediction interval becomes wider as bus headway increases. However the unweighted regression resulted in a constant prediction interval regardless of the level of bus headway. Consequently the WLS model was able to provide more realistic prediction intervals for a given bus headway.

VII. Conclusion

The models for forecasting the bus dwell time and prediction interval were developed in this study. A probabilistic model was developed and its related probability distributions were defined. A binomial distribution and a negative binomial distribution were exploited for alighting and boarding passengers, respectively. The probabilistic models are based on the linear relations between the number of alighting passengers and passenger loads, and the number of boarding passengers and bus headways. However, the observed data revealed that the number of alighting passengers is not correlated with the passenger loads. Several regression models were examined with different combination of independent variables such as bus headway, passenger loads on bus, and schedule adherence. The analysis concluded that a regression model with one independent variable, bus headway, had the best results. The residual analysis of the regression model revealed non-constant variance over bus headways, which violated the common assumption underlying
regression modeling. The weighted least squares method, therefore, was employed in order to consider the non-constant variance. Another advantage from WLS method can provide the prediction interval that varies according to the bus headway. Because of large variability in bus dwell time, the prediction interval is more informative than mean dwell time in supporting ATIS and Transit Signal Priority(TSP) systems.

The Bus Management System(BMS) and Smart Card System(SCS) established through the Seoul's Bus Reform Project in 2004 made it possible to identify the bus location and the number of passenger boarding at each stop. The real-time data from BMS and SCS increase the forecasting reliability of bus arrival time at specific stops and intersections. The methodology proposed in this study can utilize the real-time data and produce more reliable results. The parameters of the models in this study however are needed to be adjusted in order to consider the characteristics of the passenger behavior and bus networks of Seoul.

Reference


