

Computational Results and Verifications of an Analytical Dynamic User-optimal Traffic Assignment Model

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수리적 사용자동적통행배정모형의 계산적 검증

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ABSTRACT : This paper aims to provide computational results in order to verify the performance of a dynamic traffic assignment (DTA) model. The concerned dynamic traffic assignment model is formulated as a link-based variational inequality model attempting to achieve the dynamic user-optimal (DUO) state, which is the temporal generalization of Wardrop's first principle. A diagonalization algorithm is utilized to solve the model iteratively to the convergence. Computational verifications of this DTA model are performed and reported in terms of (1) attainability of the DUO state, (2) valid flow propagation, (3) maintenance of first-in-first-out (FIFO) trip ordering, and (4) model convergence. A significant contribution of this paper is that it offers to the academic community with comprehensive computational examples and verifications of a DTA model, which are comparable to similar works. For easy comparisons, a small-scale network containing seven nodes and ten links by Wie et al(1994) is adopted. Conclusions and future research needs are furnished.

Key words : DTA, FIFO, variational inequality, flow propagation, diagonalization algorithm

요약 : 동적통행배정모형(Dynamic Traffic Assignment (DTA) Model)은 첨단교통체계(ITS: Intelligent Transportation System)의 실시간 교통정보제공을 가능하게 하는 핵심요소로 인정되고 있다. 특히 수리적 동적통행배정모형(Analytical DTA Model)은 교통시스템의 최적상황의 수식화와 전략개발 수립에 대한 이론적인 장점 때문에 최근까지 활발하게 연구되고 있다. 본 연구는 수리적 동적통행배정모형에서 포함하고 있는 수학적 특성과 제약조건이 실제 교통망에 적용해서도 만족하는지에 대한 검증에 초점을 맞추고 있다. 본 연구에서 적용하는 통행배정모형은 Wardrop의 사용자 동적 최적원리를 링크기반 변동부등식(Link-Based Variational Inequality)에 근거하여 정식화되었으며, 대각화기법은 모형의 최적해 수렴여부를 검증하기 위하여 적용되었다. 모형의 계산적 검증은 (1) 동적사용자 최적 (Dynamic User Optimal), (2) 교

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통류의 동적흐름(Flow Propagation), (3) 교통류 선입선출(FIFO: First-In-First-Out) 조건, (4) 모형의 수렴 (Model Convergence)의 4가지 조건의 수행성 여부를 검토한다.

주제어 : 동적통행배정, 선입선출, 변등부등식, 교통류 전파, 대각화 알고리즘

I. Introduction

Dynamic Traffic Assignment (DTA) provides a more realistic representation of the traffic flow and resulting flow pattern than its static counterpart by considering traffic variation in temporal domain. The DTA modeling has become a core of transportation research in recent years to play this role and is gradually maturing (Carey, 1987; Friesz et al., 1993; Daganzo, 1995; Ran et al., 1997; Chen & Hsueh, 1998; Wu et al., 1998).

In the DTA models, however, there still exist important and challenging questions to any dynamic network modeling, such as attainability of the DUO state, valid flow propagation, maintenance of first-in-first-out (FIFO) trip ordering, etc. (Carey, 1986; Carey, 1987). To answer these questions, it is necessary to perform comprehensive computational studies and report detailed results to verify the performance of the interested model and solution algorithm in various aspects.

This paper aims to provide computational results using reported test problems solved by other researchers to verify the performance of

a dynamic traffic assignment model. Using the variational inequality (VI) approach (The details can be referred in Nagurney (1993)). This model is formulated as a discrete-time, link-based DTA model that seeks to achieve the DUO state. A relaxation method is then used to solve this discrete-time DTA model. In this solution algorithm, a discrete-time nonlinear programming (NLP) problem is first formulated and solved by the Frank-Wolfe method (Frank and Wolfe, 1956) during each relaxation. Unlike other algorithms for solving DTA models, this algorithm uses inflow as the only independent variable to construct the resulted NLP.

In consideration of easy comparisons, the proposed DTA model and solution algorithm are then implemented on a small-scale network that contains seven nodes and ten links. The adopted test network was first constructed by Wie et al. (1994). It is known that flow-based link travel time functions are not monotonic and convex with respect to link flow (Carey, 1992). Therefore, a modified Greenshields function is used to determine the resulting speed-density relationships and derive the link travel times. Computational

results are obtained and reported in detail from two different sets of solution scenarios, including single OD pair case and multiple OD pair case. Our results show that for each OD pair at each time interval, the actual travel times experienced by travelers departing at the same time interval are equal and minimal within minor errors, which indicate that the DUO state is achieved.

II. A Variational Inequality Model

1. Notation

The notations used in the formulation and solution algorithm are summarized below. In all the notations, superscript "rs" denotes origin-destination pair (r,s), subscripts "a" denotes link a, subscript "p" denotes route p.

$x_a(t)$ =number of vehicles on link a at time t (main problem variable).

$u_a(t)$ =inflow rate into link a at time t (main problem variable).

$v_a(t)$ =exit flow rate from link a at time t (main problem variable).

$y_a(k)$ =number of vehicles on link a at the beginning of interval k (subproblem variable).

$p_a(k)$ =inflow into link a during interval

k (subproblem variable).

$q_a(k)$ =exit flow from link a during interval k (subproblem variable).

$f^{rs}(t)$ =departure flow rate from origin r to destination s at time t (given). time t(main problem variable).

$\tau_a(t)$ =actual travel time over link a for flow entering link a at time t.

$\bar{\tau}_a(t)$ =estimated actual travel time over link a for flow entering link a at time t.

$\pi^{rs}(t)$ =minimum actual route travel time between (r,s) for flow departing at time t.

2. The Model

The formulation of a link-based DTA can be derived based on the following travel-time-based ideal DUO route choice condition, which is the temporal generalization of Wardrop's first principle (Wardrop, 1952) (Sheffi, 1985).

"If , for each OD pair at each instant of time, the actual travel times experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-time-based ideal dynamic user-optimal state".

The link-based ideal DUO route choice

conditions are expressed as following equation (Ran & Boyce, 1996):

$$\pi^{rs}(t) + \tau_a[t + \pi^{rs}(t)] - \pi^{rs}(t) \geq 0, \quad \forall a = (i, j), r \quad (1)$$

$$u_a^{rs}[t + \pi^{rs}(t)] - \tau_a[t + \pi^{rs}(t)] = 0, \quad \forall a = (i, j), r, s \quad (2)$$

$$u_a^{rs}[t + \pi^{rs}(t)] \geq 0, \quad \forall a = (i, j), r, s \quad (3)$$

The equivalent VI formulation of the link-based ideal DUO route choice conditions defined in Equations (1)~(3) can be written as Equation (4), where * denotes the DUO state.

$$\int_0^T \sum_{rs} \sum_a \{ \pi^{rs*}(t) + \tau_a[t + \pi^{rs*}(t)] - \pi^{rs*}(t) \} \cdot \{ u_a^{rs}[t + \pi^{rs*}(t)] - u_a^{rs*}[t + \pi^{rs*}(t)] \} dt \geq 0 \quad (4)$$

In this combination method, the travel time approximation procedure (relaxation) is defined as the outer iteration and the F-W procedure is defined as the inner iteration. The new algorithm for solving single-class route choice model can be summarized as follows:

3. The Algorithm

The combined method is proposed. The features of the method are, 1) the travel time

approximation procedure (relaxation) is defined as the outer iteration, 2) the F-W procedure is defined as the inner iteration, and 3) the link travel time can be expressed by inflows. The algorithm for the proposed DTA model can be summarized as follows:

In each relaxation iteration, the following terms are temporarily fixed:

- ① Actual travel time $\tau_a(n)$ in the link flow propagation constraints as $\bar{\tau}_a(n)$.
- ② Actual travel time $\tau_a[n + \pi^{rs*}(n)]$ in the VI cost term $\Omega_a^{rs*}(n)$ as $\tau_a[n + \bar{\pi}^{rs*}(n)]$.

Minimal travel times $\pi^{ri*}(n)$ as $\bar{\pi}^{ri*}(n)$ and $\pi^{rj*}(n)$ as $\bar{\pi}^{rj*}(n)$ for each link and each origin node.

Step 0 Initialization

Initialize all link flows $\{x_a^{(0)}(k)\}, \{u_a^{(0)}(k)\}, \{v_a^{(0)}(k)\}$ to zero and calculate initial time estimates $\tau_a^{(1)}(k)$. Set the outer iteration counter $l = 1$.

Step 1 Relaxation

Set the inner iteration counter $n=1$. Find a new approximation of actual link travel times: $\bar{\tau}_a^{(n)}(k) = \tau(u_a^{(*)}(k), v_a^{(*)}(k), x_a^{(*)}(k))$,

where (*) denotes the final solution obtained from the most recent inner iteration. Solve the route choice problem.

[Step 1.1] Update

Calculate $\tau_a(\bar{u}_a(1), \bar{u}_a(2), \dots, \bar{u}_a(k-1), u_a(k))$ using the travel time function.

[Step 1.2] Direction Finding

Based on $\tau_a(\bar{u}_a(1), \bar{u}_a(2), \dots, \bar{u}_a(k-1), u_a(k))$, search the minimal-cost route forward from each origin to all destinations over the physical network. Perform an all-or-nothing assignment, yielding subproblem solution $p_a^r(k)$.

[Step 1.3] Line Search

Find the optimal step size that solves the one dimensional search problem using a standard line search procedure.

[Step 1.4] Move

Find a new solution by combining $u_a^r(k)$ and $p_a^r(k)$ using the optimal step size.

[Step 1.5] Convergence Test for Inner Iteration

If n equals a pre-specified number, go to step 2; otherwise, set $n = n + 1$, go to step 1.1.

Step 2 Convergence Test for Outer Iteration

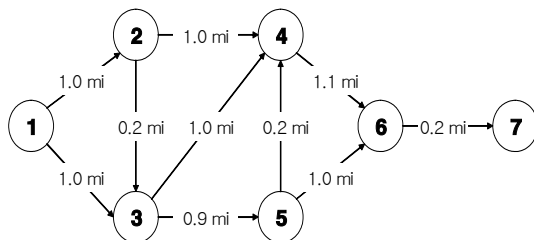
If $\bar{\tau}_a^{(l)}(k) \equiv \bar{\tau}_a^{(l-1)}(k)$, stop. The current solution $\{x_a(k)\}, \{u_a(k)\}, \{v_a(k)\}$ is in a near optimal state; otherwise, set $l = l + 1$, go to

III. Test Network and Link Travel Time Function

This section discusses the test network and link travel time function. For easy verification of computational results, a small-scale network earlier used by Wie et al. (1994) is also used in this paper. A modified Greenshields function is adopted to determine the link speed, which is a function of traffic density. Corresponding link travel times are then derived.

1. The Test Network

A hypothetical network used by Wie et al. (1994) is adopted to test the proposed model and algorithm. As shown in <Figure 1>, this network consists of ten directed links and seven nodes. Since detail network data are not presented in Wie et al. (1994), we therefore assume that all links are one-lane links with capacity of 2,200 vph and variable link length as shown in <Figure 1>.



<Figure 1> Test Network by Wie et al. (1994)

2. The Modified Greenshields Function

It is well known that using traffic flow as the variable to determine travel time does not follow a convex function with respect to flow. However, flow-based travel time functions usually provide one-to-one mapping between link travel time and flow. According to traffic flow theory, the average speed decreases as average flow increases, especially beyond the maximum flow. The flow decreases when speeds become very low, resulting in a travel time (reciprocal of speed) function that turns back and reaches high travel times (Jayakrishnan et al., 1995). However, such drawbacks are avoided if speed-density relationship is used as the basis to derive link travel times. A modified Greenshields function as shown in Equation (5) is adopted in this research to determine the link speed, which is a function of traffic density.

$$u = \begin{cases} u_{\min} + (u_{\max} - u_{\min})(1 - k/k_j) & \text{if } k \leq k_j \\ u_{\min} & \text{if } k > k_j \end{cases} \quad (5)$$

where

u = speed

u_{\min} = minimum speed at jam density

u_{\max} = free flow speed

k = density

k_j = jam density

Thus, the link travel time can be calculated by:

$$\begin{aligned} \text{if } k &\leq k_j, \\ \tau_a(t) &= \frac{L_a k_j}{u_{\min} k_j + (u_{\max} - u_{\min})(k_j - k_a(t))} \\ &= \frac{k_j L_a^2}{u_{\min} k_j L_a + (u_{\max} - u_{\min})(k_j L_a - x_a(t))} \end{aligned} \quad (6)$$

if $k > k_j$,

$$\tau_a(t) = \frac{L_a}{u_{\min}} \quad (7)$$

where

$\tau_a(t)$ = travel time on link a in time interval t

L_a = length of link a

IV. Computational Results and Analysis

Results from cases of single OD pair and multiple OD pair are obtained and analyzed in this section. Travel times on used routes are calculated from solutions of this link-based model. The achievement of DUO state is then determined if each OD pair at each time interval, the actual travel times experienced by travelers departing at the same time interval are equal and minimal within minor errors. As shown in our computational results, the OD flows propagate

smoothly across the test network along time horizon and the so-call instantaneous flow propagation is not found. An observation of FIFO violation and its preventing strategies are discussed in the last of this section.

1. Results from The Case of Single OD Pair

We first test the proposed model and algorithm on the aforementioned test network using travel demand for single OD pair (1, 7). In this solution scenario, the duration of each time interval is twenty seconds. There are twenty trips each departed from origin 1 to destination 7 in the first ten intervals. To obtain the initial solution, twenty iterations of incremental assignment are performed. As discussed in Section 2, the inner problems are solved by Frank-Wolfe algorithm. Computational results are shown in <Table 1> and <Table 2>, respectively.

<Table 1> Results from the Case of Single OD Pair (1, 7)

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
1	2	1	1.8	0	1.8	4.03	4	5
1	2	2	11.8	0	13.7	4.22	4	6
1	2	3	10	0	23.6	4.4	4	7
1	2	4	6.3	0	30	4.52	5	9
1	2	5	3.6	1.8	31.8	4.55	5	10
1	2	6	7.3	11.8	27.3	4.46	4	10
1	2	7	11.8	10	29.1	4.49	4	11
1	2	8	10	6.3	32.7	4.57	5	13
1	2	9	5.4	3.6	34.6	4.61	5	14
1	2	10	3.6	7.3	30.9	4.53	5	15
1	2	11	0	11.8	19	4.31	4	-
1	2	12	0	10	9.1	4.14	4	-
1	2	13	0	5.4	3.6	4.06	4	-
1	2	14	0	3.6	0	4	4	-
1	2	15	0	0	0	4	4	-

(continued)

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
1	3	1	18.2	0	18.2	4.3	4	5
1	3	2	8.2	0	26.3	4.45	4	6
1	3	3	10	0	36.4	4.65	5	8
1	3	4	13.7	0	50	4.94	5	9
1	3	5	16.4	18.2	48.2	4.9	5	10
1	3	6	12.7	8.2	52.7	5.01	5	11
1	3	7	8.2	5.4	55.5	5.08	5	12
1	3	8	10	9.1	56.4	5.1	5	13
1	3	9	14.6	17.3	53.7	5.03	5	14
1	3	10	16.4	18.2	51.8	4.99	5	15
1	3	11	0	10.9	41	4.74	5	-
1	3	12	0	6.3	34.6	4.61	5	-
1	3	13	0	9.1	25.5	4.43	4	-
1	3	14	0	16.4	9.1	4.14	4	-
1	3	15	0	9.1	0	4	4	-
2	4	5	1.8	0	1.8	4.03	4	9
2	4	6	11.8	0	13.7	4.22	4	10
2	4	7	10	0	23.6	4.4	4	11
2	4	8	6.3	0	30	4.52	5	13
2	4	9	3.6	1.8	31.8	4.55	5	14
2	4	10	7.3	11.8	27.3	4.46	4	14
2	4	11	11.8	10	29.1	4.5	5	16
2	4	12	10	6.3	32.7	4.57	5	17
2	4	13	5.4	3.6	34.6	4.61	5	18
2	4	14	3.6	7.3	30.9	4.53	5	19
2	4	15	0	11.8	19	4.31	4	19
2	4	16	0	10	9.1	4.14	4	-
2	4	17	0	5.4	3.6	4.06	4	-
2	4	18	0	3.6	0	4	4	-
2	4	19	0	0	0	4	4	-
3	4	8	2.7	0	2.7	4.04	4	12
3	4	9	0	0	2.7	4.04	4	-
3	4	10	3.6	0	6.3	4.1	4	14
3	4	11	2.7	0	9.1	4.14	4	15
3	4	12	0	2.7	6.3	4.1	4	-
3	4	13	0	0	6.3	4.1	4	-
3	4	14	0	3.6	2.7	4.04	4	-
3	4	15	1.8	2.7	1.8	4.03	4	19
3	4	16	0	0	1.8	4.03	4	-
3	4	17	0	0	1.8	4.03	4	-
3	4	18	0	0	1.8	4.03	4	-
3	4	19	0	1.8	0	4	4	-
3	5	5	18.2	0	18.2	3.9	4	9
3	5	6	8.2	0	26.3	4.05	4	10
3	5	7	5.4	0	31.8	4.16	4	11
3	5	8	6.4	0	38.2	4.3	4	12
3	5	9	17.3	18.2	37.3	4.28	4	13
3	5	10	14.6	8.2	43.7	4.42	4	14
3	5	11	8.2	5.4	46.4	4.48	4	15
3	5	12	6.3	6.4	46.3	4.48	4	16
3	5	13	9.1	17.3	38.2	4.3	4	17
3	5	14	16.4	14.6	40	4.34	4	18
3	5	15	7.3	8.2	39.2	4.32	4	19
3	5	16	0	6.3	32.8	4.18	4	-
3	5	17	0	9.1	23.7	4	4	-
3	5	18	0	16.4	7.3	3.72	4	-
3	5	19	0	7.3	0	3.6	4	-
4	6	9	1.8	0	1.8	4.43	4	13
4	6	10	11.8	0	13.7	4.62	5	15
4	6	11	10	0	23.6	4.79	5	16
4	6	12	9.1	0	32.7	4.96	5	17
4	6	13	3.6	1.8	34.5	5	5	18
4	6	14	10.9	6.3	39.1	5.09	5	19
4	6	15	14.6	10	43.6	5.19	5	20
4	6	16	10	9.1	44.5	5.2	5	21

(Table 1) continued)

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
4	6	17	5.4	8.2	41.8	5.15	5	22
4	6	18	3.6	2.7	42.7	5.17	5	23
4	6	19	1.8	10.9	33.6	4.98	5	24
4	6	20	0	14.6	19	4.71	5	-
4	6	21	0	10	9.1	4.54	5	-
4	6	22	0	5.4	3.6	4.46	4	-
4	6	23	0	1.8	1.8	4.43	4	-
4	6	24	0	1.8	0	4.4	4	-
5	6	9	18.2	0	18.2	4.3	4	13
5	6	10	8.2	0	26.3	4.45	4	14
5	6	11	5.4	0	31.8	4.55	5	16
5	6	12	6.4	0	38.2	4.68	5	17
5	6	13	17.3	18.2	37.3	4.66	5	18
5	6	14	14.6	8.2	43.7	4.8	5	19
5	6	15	8.2	5.4	46.4	4.86	5	20
5	6	16	6.3	4.5	48.2	4.9	5	21
5	6	17	9.1	10.9	46.4	4.86	5	22
5	6	18	16.4	18.2	44.6	4.82	5	23
5	6	19	7.3	12.7	39.2	4.7	5	24
5	6	20	0	6.3	32.8	4.57	5	-
5	6	21	0	5.4	27.4	4.47	4	-
5	6	22	0	10.9	16.4	4.27	4	-
5	6	23	0	12.7	3.7	4.06	4	-
5	6	24	0	3.7	0	4	4	-
6	7	13	20	0	20	1.98	2	15
6	7	14	14.5	0	34.5	2.39	2	16
6	7	15	15.5	20	30	2.24	2	17
6	7	16	13.6	14.5	29.1	2.21	2	18
6	7	17	19.1	15.5	32.7	2.33	2	19
6	7	18	20.9	13.6	40	2.59	3	21
6	7	19	23.7	19.1	44.6	2.79	3	22
6	7	20	20.9	18.1	47.4	2.92	3	23
6	7	21	15.4	20.9	41.9	2.67	3	24
6	7	22	16.4	23.7	34.6	2.39	2	24
6	7	23	14.6	18.2	30.9	2.27	2	25
6	7	24	5.5	16.4	20.1	1.98	2	26
6	7	25	0	14.6	5.5	1.69	2	-
6	7	26	0	5.5	0	1.6	2	-

(Table 2) Route Travel Times for OD Pair (1, 7) in Each Time Interval

	t=1	t=2	t=3	t=4	t=5
1-3-5-6-7	14.48	15.33	15.95	16.47	16.91
1-2-4-6-7	14.46	15.30	15.80	16.95	16.89
1-3-4-6-7	----*	----*	15.98	----*	16.88
	t=6	t=7	t=8	t=9	t=10
1-3-5-6-7	17.28	17.13	16.64	16.46	15.99
1-2-4-6-7	16.81	16.61	16.62	16.10	15.81
1-3-4-6-7	17.26	----*	----*	----*	15.98

* : Route is not used in corresponding time interval.

For each link in the network, (Table 1) reports results including entering interval of

trips, inflow, outflow, link flow, link travel time (in the unit of time interval), traversed time intervals, and exiting interval of trips in corresponding time intervals. For example, results in the first row of (Table 1) tell that there are 1.8 trips entering link (1, 2) in interval 1. It took 4.03 time intervals for those 1.8 trips to traverse link (1, 2). Thus, the traversed time intervals are 4 after round-off. As a result, these 1.8 trips didn't exit this link until interval 5 (see row 1, column 8 in (Table 1)). Similarly, there are 31.8 trips on link (2, 4) in the end of interval 9. Additional 7.3 trips entered link (2, 4) and 11.8 trips (entered this link in interval 6) exited this link in interval 10. To this end, 27.3 trips remained on link (2, 4) in the end of interval 10. In this model, trips won't exit a specific link only if those trips have traversed on that link for certain time intervals that they should experience. This prevents the occurrence of so-called instantaneous flow propagation from this model and resulting solutions. Therefore, OD flows propagate across the network in order. According to the relationships between the second column and the eighth column in (Table 1), the first-in-first-out trip ordering is clearly maintained in the computational results.

(Table 2) shows the route travel times for OD pair (1, 7) in each time interval. Because this is a link-based model, travel times on

used routes are obtained by adding up travel times on corresponding links. For example, for those trips departed node 1 (origin, and node 7 is the destination) in interval 1, the travel time on route 1-3-5-6-7 is obtained by adding up travel times on links (1, 3), (3, 5), (5, 6), and (6, 7) in corresponding time intervals. The resulting travel time on this used route is 14.48 ($4.03+3.90+4.30+1.98=14.48$). The satisfaction of ideal DUO state is verified if each OD pair at each time interval, the actual travel times experienced by travelers departing at the same time interval are equal and minimal within minor errors as described previously. Since this model is solved using discrete time intervals, the route travel times may be affected by round errors. If this factor (round errors) is taken into consideration, the resulting route travel times between OD pair (1, 7) are viewed as equal, and the ideal DUO state defined in Section 2 is achieved. For example, as shown in Table 2, travel times on used routes of OD pair (1, 7) in time interval 5 are 16.91, 16.89, and 16.88 (time intervals), respectively; which can be viewed as the same if we round them to the nearest integer 17.

2. Results form The Case of Multiple OD Pairs

The proposed model and algorithm is

tested on the aforementioned test network using travel demand for multiple OD pairs including (1, 7), (2,6), and (3, 7). The duration of each time interval is still twenty seconds. In this solution scenario, there are twenty trips each departed from origins to destinations in the first ten intervals. To obtain the initial solution, twenty iterations of incremental assignment are performed. The inner problems are solved by Frank-Wolfe algorithm. Computational results of this solution scenario are shown in <Table 3> and <Table 4>, respectively.

<Table 3> Results from the Case of Multiple OD Pairs

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
1	2	4	5	0	5	4.08	4	8
1	2	5	1	0	6	4.09	4	9
1	2	6	3	0	9	4.14	4	10
1	2	7	4	0	13	4.21	4	11
1	2	8	6	5	14	4.23	4	12
1	2	9	9	1	22	4.37	4	13
1	2	10	7	3	26	4.44	4	14
1	2	11	0	4	22	4.37	4	-
1	2	12	0	6	16	4.26	4	-
1	2	13	0	9	7	4.11	4	-
1	2	14	0	7	0	4	4	-
1	3	1	20	0	20	4.33	4	5
1	3	2	20	0	40	4.72	5	7
1	3	3	20	0	60	5.19	5	8
1	3	4	15	0	75	5.61	6	10
1	3	5	19	20	74	5.58	6	11
1	3	6	17	15	76	5.64	6	12
1	3	7	16	15	77	5.67	6	13
1	3	8	14	16	75	5.61	6	14
1	3	9	11	17	69	5.43	5	14
1	3	10	13	19	63	5.27	5	15
1	3	11	0	17	46	4.85	5	-
1	3	12	0	15	31	4.54	5	-
1	3	13	0	13	18	4.3	4	-
1	3	14	0	10	8	4.13	4	-
1	3	15	0	8	0	4	4	-
2	3	8	1	0	1	1.62	2	10
2	3	9	3	0	4	1.66	2	11
2	3	10	5	1	8	1.73	2	12
2	3	11	0	3	5	1.68	2	-
2	3	12	0	5	0	1.6	2	-
2	4	1	20	0	20	4.33	4	5
2	4	2	20	0	40	4.72	5	7
2	4	3	20	0	60	5.19	5	8
2	4	4	20	0	80	5.76	6	10

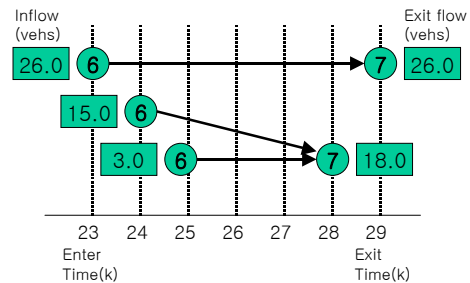
(<Table 3> continued)

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
2	4	5	20	20	80	5.76	6	11
2	4	6	20	15	85	5.92	6	12
2	4	7	20	15	90	6.1	6	13
2	4	8	24	18	96	6.32	6	14
2	4	9	18	18	96	6.32	6	15
2	4	10	18	20	94	6.24	6	16
2	4	11	4	20	78	5.7	6	17
2	4	12	6	20	64	5.29	5	17
2	4	13	9	22	51	4.97	5	18
2	4	14	7	20	38	4.68	5	19
2	4	15	0	12	26	4.44	4	-
2	4	16	0	7	19	4.31	4	-
2	4	17	0	9	10	4.16	4	-
2	4	18	0	9	1	4.02	4	-
2	4	19	0	1	0	4	4	-
3	4	3	6	0	6	4.09	4	7
3	4	4	8	0	14	4.23	4	8
3	4	5	9	0	23	4.39	4	9
3	4	6	9	0	32	4.56	5	11
3	4	7	6	6	32	4.56	5	12
3	4	8	12	8	36	4.64	5	13
3	4	9	12	9	39	4.7	5	14
3	4	10	12	9	42	4.76	5	15
3	4	11	6	6	42	4.76	5	16
3	4	12	5	6	41	4.74	5	17
3	4	13	1	15	27	4.46	4	17
3	4	14	1	13	15	4.24	4	18
3	4	15	4	7	12	4.19	4	19
3	4	16	0	5	7	4.11	4	-
3	4	17	0	2	5	4.08	4	-
3	4	18	0	1	4	4.06	4	-
3	4	19	0	4	0	4	4	-
3	5	1	20	0	20	3.93	4	5
3	5	2	20	0	40	4.34	4	6
3	5	3	14	0	54	4.67	5	8
3	5	4	12	0	66	5	5	9
3	5	5	31	20	77	5.35	5	10
3	5	6	26	20	83	5.56	6	12
3	5	7	29	14	98	6.16	6	13
3	5	8	24	11	111	6.81	7	15
3	5	9	25	17	119	7.27	7	16
3	5	10	28	29	118	7.21	7	17
3	5	11	14	24	108	6.65	7	18
3	5	12	15	21	102	6.35	6	18
3	5	13	12	32	82	5.52	6	19
3	5	14	9	26	65	4.97	5	19
3	5	15	4	14	55	4.7	5	20
3	5	16	0	15	40	4.34	4	-
3	5	17	0	20	20	3.93	4	-
3	5	18	0	15	5	3.68	4	-
3	5	19	0	5	0	3.6	4	-
4	6	5	20	0	20	4.73	5	10
4	6	6	15	0	35	5.01	5	11
4	6	7	21	0	56	5.46	5	12
4	6	8	26	0	82	6.15	6	14
4	6	9	27	7	102	6.81	7	16
4	6	10	29	17	114	7.28	7	17
4	6	11	26	14	126	7.82	8	19
4	6	12	26	20	132	8.12	8	20
4	6	13	37	17	152	9.32	9	22
4	6	14	33	29	156	9.6	10	24
4	6	15	19	24	151	9.25	9	24
4	6	16	12	25	138	8.45	8	24
4	6	17	11	22	127	7.87	8	25
4	6	18	10	22	115	7.32	7	25

(continued)

From Link	To Link	Ent. Int.	In flow	Exit flow	Link flow	Link time (τ_a)	Int (τ_a)	Exit int.
4	6	19	5	19	101	6.78	7	26
4	6	20	0	24	77	6.01	6	-
4	6	21	0	26	51	5.35	5	-
4	6	22	0	24	27	4.86	5	-
4	6	23	0	15	12	4.59	5	-
4	6	24	0	9	3	4.45	4	-
4	6	25	0	3	0	4.4	4	-
5	6	5	20	0	20	4.33	4	9
5	6	6	20	0	40	4.72	5	11
5	6	7	14	0	54	5.04	5	12
5	6	8	11	0	65	5.32	5	13
5	6	9	17	20	62	5.24	5	14
5	6	10	29	15	76	5.64	6	16
5	6	11	24	15	85	5.92	6	17
5	6	12	21	12	94	6.24	6	18
5	6	13	32	15	111	6.94	7	20
5	6	14	26	15	122	7.49	7	21
5	6	15	14	27	109	6.85	7	22
5	6	16	15	20	104	6.64	7	23
5	6	17	20	20	104	6.64	7	24
5	6	18	15	28	91	6.13	6	24
5	6	19	5	25	71	5.49	5	24
5	6	20	0	19	52	4.99	5	-
5	6	21	0	13	39	4.7	5	-
5	6	22	0	13	26	4.44	4	-
5	6	23	0	18	8	4.13	4	-
5	6	24	0	8	0	4	4	-
6	7	9	20	0	20	1.98	2	11
6	7	10	15	0	35	2.4	2	12
6	7	11	15	20	30	2.24	2	13
6	7	12	18	15	33	2.34	2	14
6	7	13	19	15	37	2.47	2	15
6	7	14	27	18	46	2.85	3	17
6	7	15	34	19	61	3.83	4	19
6	7	16	29	22	68	4.56	5	21
6	7	17	26	25	69	4.69	5	22
6	7	18	32	30	71	4.97	5	23
6	7	19	30	31	70	4.83	5	24
6	7	20	31	25	76	5.83	6	26
6	7	21	30	28	78	6.27	6	27
6	7	22	30	30	78	6.27	6	28
6	7	23	26	28	76	5.83	6	29
6	7	24	15	26	65	4.22	4	28
6	7	25	3	30	38	2.51	3	28
6	7	26	0	18	20	1.98	2	-
6	7	27	0	16	4	1.66	2	-
6	7	28	0	4	0	1.6	2	-

The shaded areas denote the violations of FIFO



<Figure 2> FIFO Violation

〈Table 4〉 Route Travel Times for OD Pairs in Each Time Interval

	t=1	t=2	t=3	t=4	t=5
1-3-5-6-7	19.88	23.66	25.12	23.67	22.57
1-3-4-6-7	20.09	23.23	25.41	23.84	23.01
1-2-4-6-7	----*	----	----	24.21	23.88
2-4-6	9.06	10.18	11.34	13.04	13.58
2-3-4-6	----	----	----	----	----
3-5-6-7	10.24	11.30	12.47	13.10	15.55
3-4-6-7	----	----	11.89	13.23	15.75
	t=6	t=7	t=8	t=9	t=10
1-3-5-6-7	22.33	20.89	20.28	20.11	17.47
1-3-4-6-7	20.76	20.51	19.68	19.51	18.21
1-2-4-6-7	23.05	20.29	19.90	19.17	17.87
2-4-6	14.05	15.41	15.92	15.57	14.69
2-3-4-6	----	----	15.46	14.44	14.21
3-5-6-7	16.77	18.94	19.93	19.74	18.06
3-4-6-7	17.20	18.51	20.22	18.52	18.23

* : Route is not used in corresponding time interval.

Similar to results shown in 〈Table 1〉, 〈Table 2〉 also reports results including: entering interval of trips, inflow, outflow, link flow, link travel time (in the unit of time interval), traversed time intervals, and exiting interval of trips for each link in corresponding time intervals. By checking the relationships between the second column (entering interval of trips) and the eighth column (exiting intervals of trips) in 〈Table 2〉, it is clear that there is no instantaneous flow propagation in this solution example.

In this link-based dynamic traffic assignment model, by summing up travel times on corresponding links leads to route travel times. 〈Table 2〉 shows the used route travel times for all OD pairs in each time

interval. The satisfaction of ideal DUO state is again verified if each OD pair at each time interval, the actual travel times experienced by travelers departing at the same time interval are equal and minimal within minor errors. If the factor of rounding error is taken into consideration, the resulting travel times on used routes between all OD pairs in this solution example are actually fairly equal, indicating the achievement of ideal DUO state defined in Section 2.

3. Discussions of FIFO Violations and Prevention

FIFO assumption is an approximation of reality and it may not occur in actuality. However, FIFO is known that should be maintained when there is only one lane and no extra spaces for turning movements at intersections. When FIFO is violated, the overtaking occurs. Overtaking denotes a late entering vehicle propagates faster and exits earlier than an earlier entering vehicle. Overtaking violates the FIFO rule for traffic propagation on links, although it might happen on two-lane links. In 〈Table 3〉, FIFO (first-in-first-out) violation occurs on link (6, 7) at intervals 24 and 25. It is identified that traffic entering link (6→7) at time intervals 24 and 25 exits earlier than traffic entering link at time interval 23. The detail explanation

on this phenomenon is illustrated at <Figure 2>.

Denote the link travel time for flows entering link a at time t as $\tau_a(t)$. The travel time for flows entering link a at time $t + \Delta t$ is $\tau_a(t + \Delta t)$. If we require that overtaking should not occur, we must allow the clock time $t + \tau_a(t)$, when flows entering at time t must exit link a , to be smaller than the clock time $t + \Delta t + \tau_a(t + \Delta t)$, the exiting time for flows entering link a at time $t + \Delta t$. It follows that (Ran and Boyce, 1996):

$$t + \tau_a(t) < t + \Delta t + \tau_a(t + \Delta t) \quad (8)$$

Dividing the above equation by Δt , we obtain

$$\frac{\tau_a(t + \Delta t) - \tau_a(t)}{\Delta t} > -1 \quad (9)$$

The above condition must be met to avoid overtaking in any dynamic route choice model using link travel time functions in the flow propagation constraint. If the decreasing rate of travel time on any link a exceeds 1, overtaking will occur. However, there is a possibility for FIFO violations if travel times change so rapidly. To avoid FIFO violations, a proper link travel time and interval length should be concerned. It is known not easy to choose proper link travel times. As expressed

in Equation (10), link travel time is a function of link flow, and link flow can be expressed as Equation (11). If in Equation (12) the difference of link flows between two consecutive intervals rapidly turns to be negative then it causes FIFO violations.

$$\tau_a(t) = f(x_a(t)) \quad (10)$$

$$x_a(t) = x_a(t-1) + u_a(t) - v_a(t) \quad (11)$$

$$x_a(t) - x_a(t-1) = u_a(t) - v_a(t) \quad (12)$$

Incorporating link capacity constraints and/or changing the network structure, for example, introducing dummy nodes or artificial links, can prevent the FIFO violation.

V. Concluding Remarks and Future Research

In order to verify the performance properties of a dynamic traffic assignment (DTA) model, such as (1) attainability of the DUO state, (2) valid flow propagation, (3) maintenance of first-in-first-out (FIFO) trip ordering, and (4) model convergence, evident computational results are provided in this paper. The conclusions include:

- 1) The analytical DTA model is formulated as a Variational Inequality (VI) and can be solved efficiently to convergence by

the proposed relaxation algorithm. Also DUO state is guaranteed.

- 2) FIFO propagation is kept for most cases. Specific solutions are proposed for the occasionally FIFO-violated cases.

With the strict theoretical considerations and the convincing computational results, the analytical DTA model is proved to be appropriate to be applied in the real-time traffic prediction and traffic control. Enhancements of analytical DTA model for real time applications, such as rolling horizon implementation, traffic control model, and on-line calibration, are the major future research directions.

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