Optimal Public Facility Location Model with Congestion Effects and Travel Costs

I. Introduction

Distinctive characteristics of public goods(services)[1] as Samuelsonian pure public goods, are nonrivalry and nonexcludability. Most public services, however, are not pure public in nature, that is they can be excludable and congestible.

Generally, congestion in public services refers to the decreasing effective service levels, holding public sector resources fixed, as the size of the consumers increases. Since welfare of the users is highly diminished by congestion effects such as crowding and waiting time, incorporation of this property in determining the optimal facility size and location is very important. However, most studies on congestion effects have concerned with optimal pricing or taxing of the services based on the public sector economics (for example, Hochman(1982), Haveman et al(1977), Brueckner(1981), Craig(1987)). Little attention has been placed on the optimal size and locations of facilities with congestion effects (see, Schuler et al. (1976)).

This paper tries to develop a model for the optimal facility locations with both congestion effects and transportation costs on a network. Section 2 is a review of some of the literature on the theory of optimal provision of (congestible) public services. In section 3.1, a marginal congestion cost is derived through a traditional partial equilibrium utility function. In section 3.2, a Hakimi-type m-median model is modified to get a welfare loss function due to travel distances on a network. In section 4, two previous formulations are combined and a model of optimal locations of public facilities, with both congestion and travel costs, on a network is presented. The last section includes a summary and limitations of this paper.

II. Congestion in public services

In the partial equilibrium analysis of a pure public good, the optimal quantity of the public good is found at the intersection point of the total demand curve[2] and the supply curve. In other words, pure public good equilibrium is established where the total willingness to pay for the public good is equal to the price at which a producer is willing to supply that level of output(Brown and Jackson, 1978). In the general equilibrium analysis of a pure public good, the optimal provision requires that the sum of consumers' marginal rates of substitution between any public good and private good equal their marginal rates of transformation in production. Formally state, $\text{SUM from} \{i\} \text{ MRS}^{i} = \text{MRT}$, where i is a consumer (Samuelson 1955, cited in Herber 1979). However, the optimal provision of (partially) congestible public services has received far less attention, even though most public services are subject to congestion effects.

Hochman(1982) classified impure local public goods into two types: congestible and pollutable. Focused on the first, a version of Samuelson's rule as to the optimal allocation of pure public goods is extended on the one hand to locally dispersed public goods and on the other hand to congestible local public goods. Two corrective Pigovian taxes[3] are identified: congestion tolls levied on households, and a residential land tax. These two taxes cover total government expenditure on the local public goods. While, Haveman et al.(1977) analyzed congestion as a consumption externality in a model in which the market demand of consumers of facility services reflects heterogeneous tastes for congestion avoidance. The individual willingness to pay is a function of the aggregate level of facility use. The analysis demonstrated that the Pareto optimal toll for facility use depends on the pattern by which congestion cost is distributed among facility users.

Brueckner(1981) estimated the strength of congestion effects for fire protection services through a community's fire insurance rating. The empirical results show that the congestion properties of fire protection are much like those of a pure public good. In addition, a new notion of returns to scale for public goods is introduced and relevant parameters for the fire protection case are estimated.

Bergstrom and Goodman(1973) and Borcherding and Deacon(1972) attempted to estimate the strength of congestion for various public goods. Both papers postulated that the consumption level of a public good equals Xn^{r}, where X is a measure of public output, n is the size of the consuming group, and r is the congestion parameter(r 0). Both papers further assumed that as a result of majority voting process, public good consumption in a community is set at the level of desired by the median income voter. A public good demand function was then used to derive an estimating equation relating public expenditure to a community's median income level, population, and other variables. The congestion parameter r was estimated by a nonlinear function of the

regression coefficients from this equation(cited in Brueckner, 1981, 45-6).

Schuler and Holahan(1976) developed a maximum covering location model to find out an optimal size and spacing of public facilities under the assumption of continuous space and uniform population density. After classifying various public services with their congestibility, congestion effects were included in the model as a function of the size of the population served. The analysis suggests the potential for multiple local optimum combinations of service area and facility size when the congestion effects are considered.

In the use of impure public services, the welfare of user may depend positively upon the quantity of services consumed and negatively on 'congestion', where congestion is a function of the size of the facility[4] and the total use of it by all users(Oakland, 1972). Given a fixed facility size, the effective service level per capita (W) is a concave function of number of users (n) of the facility, in other words, $dW/dn<0$ and $d^2W/dn^2<0$.[5] This property can also be applied in the spatial friction case due to the distances between demand and supply nodes.

III. Congestion Effects and Travel Costs in the Public Facility Location

Assume that there is only one kind of congestible public facilities on a network. People consume the services that are provided from the facilities to maximize their welfare. The actual service levels will be diminished due to the additional use of the services. Given n nodes on a network that can be demand (X_i) or supply node (X_j), number of facilities provided on the network is limited because of public sector resource limitation(or budget constraint). The total number of facilities to be allocated (m) is less than the number of nodes (n) (m<n). Some nodes should be allocated to a supply node to meet their demands. Users in any node (except, supply nodes) should travel to use the services from the allocated facility in a supply node resulting welfare loss due to travel costs. Consumers can use the services with variable intensities. Let Y(endogenous) be the number of visits of each user (per week) and a_i be the number of users in the *i*th node.

3.1 Marginal Congestion Cost

From the traditional utility function, marginal congestion cost can be derived.

Within the partial equilibrium analysis, for simplicity, the social welfare can be expressed as a function of size of the facility, number of visits, and number of users for the services.

$$W(S, a_iY, Y) \tag{1}$$

where, W is social welfare, S is the size of the public facility(endogenous). Users benefit from the size of the facility and number of visits, but suffer a loss of welfare due to extra total use. Therefore, it is assumed that $\partial W/\partial S > 0$, $\partial W/\partial a_iY < 0$, and $\partial W/\partial Y > 0$.

Assume that total costs for providing the services consist of facility costs and operation costs. Then the production capacity of the public sector for the services is given by the function:[6]

$$F(mS, a_iY) = B \tag{2}$$

where, B is budget constraint, a_iY is total use.

The optimal size of the facilities and number of visits are obtained by maximizing social welfare (1), subject to the budget constraint (2). Reformulating the previous functions as a Lagrangian function, the optimization model is,

$$Z = W(S, a_iY, Y) + \lambda(B - F(mS, a_iY)) \tag{3}$$

where, λ is the Lagrangian multiplier.

The first order conditions reduce to the following simultaneous equations:

$$Z_\lambda = B - F(mS, a_iY) = 0 \tag{4}$$

$$Z_s = W_s - \lambda mF_s = 0 \tag{5}$$

$$Z_{aiY} = W_{aiY} - \lambda F_{aiY} = 0 \tag{6}$$

$$Z_Y = W_Y - \lambda a_iF_Y = 0 \tag{7}$$

From the equation (5) and (6),

$$W_s / mF_s = W_{aiY} / F_{aiY} = \qquad (8)$$

where W_s= PARTIAL W/ PARTIAL S, F_s= PARTIAL F/ PARTIAL S, W_{aiy}= PARTIAL W/ PARTIAL aiY, and F_{aiy}= PARTIAL F/ PARTIAL aiY.

Since   PARTIAL W/ PARTIAL aiY<0,   the sign of W_{aiY} is <u>negative</u>.

Equation (8) implies that the marginal congestion cost(welfare loss) imposed on existing users by the addition of one more user must equal the marginal social welfare increase (in terms of the marginal cost by one unit service increase). The optimal facility size and number of visits per user can also be obtained from the simultaneous equations (5), (6) and (7). From the equation (8), <u>marginal congestion cost function</u> is,

$$W_{aiY} = (W_s * F_{aiY}) / mF_s \qquad (9)$$

3.2 Travel Costs Minimizing Function

The allocation of demands to facilities greatly depend on the number and location of facilities. Given the budget constraint, number of facilities is fixed. Location of facilities determine the optimal allocation of demands. Following the multiple facilities minisum criteria on a network, travel cost minimization function can be stated as follows:

$$\text{SUM from } \{ i \} \text{ SUM from } \{ j \} \qquad a_i Y \ d_{ij}x_{ij}$$
(10)

where d_{ij} is shortest distance[7] (miles) from node i to node j (supply node),   is per mile welfare loss index, $x_{ij} = 0$ if node i does not assign to node j, 1 if node i does assign to node j

In order to convert a travel cost function to the welfare diminution function, and Y are included. Each node should be fully assigned to meet each node's demand,

$$\text{SUM from } \{ j \} \quad x_{ij}=1 \qquad i = 1,2,3, \ldots\ldots\ldots, n \qquad (11)$$

To avoid mutual assignment or relay assignment between the nodes:

$$x_{jj} \geq x_{ij} \qquad (8) \qquad\qquad\qquad (12)$$

$$i = 1,2,3, ..., n$$
$$j = 1,2,3, ..., n$$
$$i \neq j$$

Number of facilities is the same with number of nodes which assign to themselves,

$$\text{SUM from } \{ i \} \quad x_{ii} = m \qquad\qquad\qquad (13)$$

The objective of this function is to measure a loss of social welfare due to travel distances. In this formulation, it is assumed that:

> Users of a node are concentrated to the center of the node. Thus if node i is the site of a facility location, the users of node i travel zero miles to the facility.
> Each node must be allocated a supply node to satisfy the demand of node. Each node will be assigned to the nearest facility, which may or may not be located within the node.
> Facilities cannot be partially provided to serve a fraction of a node's users. That is facilities should be fully built or not at all.

IV. Optimal Public Facility Location Model with Congestion Effects and Travel Costs

Using the previous formulations in section III, optimal public facility location and demands allocation function with both congestion costs and travel costs is reduced:

$$\text{Min } Z = C + T \qquad\qquad\qquad (14)$$

where C is total congestion costs, and T is total travel costs. From equation (9) and (10), objective function[9] to be minimized is:

$$\text{Min } Z = \sum_{i} \sum_{j} -W_{aiY} a_i X_{ij} + \sum_{i} \sum_{j} a_i Y\, d_{ij} X_{ij} \qquad (15)$$

$$\text{s. t} \qquad W_{aiY} = (W_s * F_{aiY})/mF_S$$

$$\sum_{j} X_{ij} = 1 \qquad\qquad i = 1,2,3, \dots\dots\dots\dots, n$$

$$X_{jj} \quad X_{ij} \qquad\qquad i = 1,2,3, \dots\dots\dots\dots, n$$

$$j = 1,2,3, \dots\dots\dots\dots, n$$

$$i \quad j$$

$$\sum_{i} X_{ii} = m$$

By minimizing the objective function with given constraints and the nonnegativity constraints, we may get the optimal locations of facilities on the network. For this integer programming, enumeration method or heuristic approaches such as Maranzana's node partitioning, myopic approach, and node substitution can be used.[10] There might arise fractional assignments solutions. In order to find the optimal integer solution, branch and bound method can be applied.

V. Concluding Remarks

Despite the importance of congestion effects on welfare of public facility users, most public facility location-allocation modeling have neglected the congestion effects. This paper is an attempt to develop a model to consider congestion effects as well as traveling costs, incorporating a traditional utility function and a travel cost minimization programming. This formulation may be applied in location of public clinics and public social service agencies such as district offices of the Department of Public Social Services where public resources may be limited and congestion effects could be significant.

Since my effort is much directed to conceptual model building, the future researches for operation of the model should answer these following limitations:

> How can we effectively measure individual welfare or derive an individual welfare function?

> Does simple aggregation of individual welfare become a social welfare function?

> How could we quantify marginal welfare loss due to congestion?

> Does travel cost function monotonically increase by distance?

> Is it possible to combine an Integer Linear Programming with a congestion cost function?

<u>Note</u>

1) Services and goods both create utility or satisfy a want (Webster's Third New International Dictionary). For simplicity, in this paper, services are treated as intangible goods, and goods and services are used interchangeably.

2) In this case, it is assumed the total demand curve is the vertical sum of individual demand due to equal availability of the public good to everyone. It must also be assumed that each person accurately reveals her/his willingness to pay for the output of the public good(Brown and Jackson, 1978, 40-53).

3) Pigovian policy suggests that a government subsidizes to encourage production if the good produces a positive externality. If the good creates negative externality, government discourages its production through the imposition of a tax(Herber, 1979, 36-43).

4) Here, a facility means a source of club services that are simultaneously consumed by many users but are subject to congestion. The more people who use the facility, the less is the welfare each one obtains from it. This notion of a club good was originally coined by Buchanan(Buchanan 1965, 1-14).

5) In the case of private services, convex relationships between service levels and number of users are assumed caused by complete congestibility, in other words, $dW/dn<0$, and $d^2W/dn^2>0$. While in pure public services, effective service levels may be constant regardless of user size. Theoretically, a welfare loss due to congestion effects in impure public services can be explained by <Figure 1>. Let W is individual welfare, N is number of users. Let $W=f(n)=en^2+bn+c$. Then aggregate welfare loss due to congestion effects will be $a=cn-\int_{0}^{n}(en^2+bn+c)dn$. Welfare diminution is also occurred through travel costs. In <Figure 2>, D is distance from a facility, W is welfare level, d is the service threshold. In this case, W and T(travel costs) both are functions of distance from the origin of the facility. Then the aggregate net welfare at d^* is $\int_{0}^{d^*}W(d)dd-\int_{0}^{d^*}T(d)dd$.
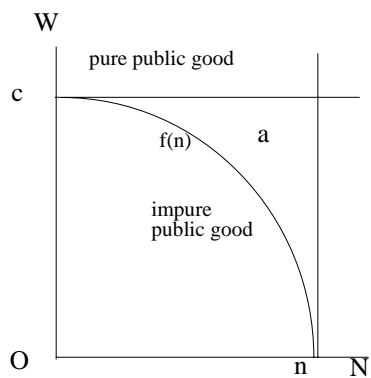
<Figure 1> Congestion Effects
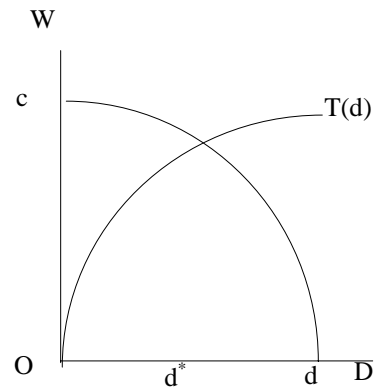
<Figure 2> Transportation Effects

6) Compare this with the transformation function in general equilibrium analysis(see, Boadway 1980, 131-7).

7) When travel is restricted to take place on a network, the distance between two nodes is used the shortest distance. Other important distance measures include Euclidean distance and rectilinear distance(see, Handler and Mirchandani, 1979, 3-4).

8) For a detailed discussions, see ReVelle and Swain, 1970, 33.

9) See Mueller(1989), for more detailed explanation of the welfare loss and additive nature of the social welfare function.

10) For more details, see Handler and Mirchandani 1979, 47-70, Kaufman 1977, 26-83, Plane and McMillan 1971, chap. 3, Taha 1981, 41-70.

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